# CORRECTION TO “MINIMAL UNIT VECTOR FIELDS”

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The paper “Minimal unit vector fields” by O. Gil-Medrano and E. Llinares-Fuster [3]. is a seminal paper in the field that has been cited by many authors – see, for example, [1, 2, 4, 5, 6, 7] to give just a few of the recent citations. It contains, however, a minor technical mistake in Theorem 14 that is important to fix. In this short note, we will provide a correction to that result. We begin by establishing the requisite notation. Let $(M, g)$ be a Riemannian manifold Assume that there exists a unit Killing vector field $V$ on $M$. Let $\nabla$ be the Levi-Civita connection of $g$. Set:

$$L_V = \mathrm{id} + (\nabla V)^t \circ \nabla V, \qquad f(v) := \sqrt{\det(L_V)},$$
$$K_V := f(V) L_V^{-1} \circ (\nabla V)^t, \quad \omega_V(X) := \mathrm{Tr}(Z \to \nabla_Z K_V(X))\,.$$

Since $V$ is a Killing vector field, the rank of $\nabla V$ must be even. We further normalize the choice of frame so that $\nabla V(E_i) = -\lambda_i E_{i^\star}$ and $\nabla V(E_{i^*}) = \lambda_i E_i$ for $i \in \{1, ..., m\}$ and $\nabla V(E_\alpha) = 0$ for $2m+1 \leq \alpha \leq n$. Thus we can take our frame to be:

$$\{E_1, E_{1^\star}, ..., E_m, E_{m^\star}, ..., E_n = V\}\,.$$

We adopt the sign convention for the curvature given in [3], namely:

$$R(x, y, z, w) = -g((\nabla_x \nabla_y - \nabla_y \nabla_x - \nabla_{[x,y]})z, w)\,.$$

In [3] the authors stated as Theorem 14 the following result:

**Theorem 1.** *Let $V$ be a unit Killing vector field, then $\omega_V = f\tilde{\rho}_V$, where $\tilde{\rho}_V(X)$ is defined to be*

$$\sum_{j=1}^{n} (R((L_V^{-1} \circ \nabla V)(X), (L_V^{-1} \circ \nabla V)(E_j), V, E_j) + R(L_V^{-1}(X), L_V^{-1}(E_j), V, E_j)).$$

*Consequently, $V$ is minimal if and only if the 1-form $\tilde{\rho}_V$ annihilates $\mathcal{H}^V$.*

Unfortunately, this is not quite correct. The correct result is as follows:

**Theorem 2.** *Let $V$ be a unit Killing vector field, then*

$$\omega_V(X) = f\tilde{\rho}_V(X) - ((L_V^{-1} \circ \nabla V)X)f\,,$$

*where $\tilde{\rho}_V(X)$ is defined to be*

$$\sum_{j=1}^{n} (R((L_V^{-1} \circ \nabla V)(X), (L_V^{-1} \circ \nabla V)(E_j), V, E_j) - R(L_V^{-1}(X), L_V^{-1}(E_j), V, E_j)).$$

*Consequently, $V$ is minimal if and only if $f\tilde{\rho}_V(X) = ((L_V^{-1} \circ \nabla V)X)f$ for any vector field $X$ orthogonal to $V$.*

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*Proof.* From the proof of Theorem 14 [3], we obtain

$$\begin{aligned}\frac{1}{f}\omega_V(E_i) =& \frac{2\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}E_{i^*}(\lambda_j) + \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_{i^*}(\lambda_i) \\ &+ \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{n-1}G_{ji^*}^{j} + \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{ji}^{j^*} - G_{j^*i}^{j}),\end{aligned} \tag{1.1}$$

$$\begin{aligned}\frac{1}{f}\omega_V(E_{i^*}) =& -\frac{2\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}E_{i}(\lambda_j) - \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_{i}(\lambda_i) \\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{n-1}G_{ji}^{j} - \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{j^*i^*}^{j} - G_{ji^*}^{j^*}),\end{aligned} \tag{1.2}$$

$$\frac{1}{f}\omega_V(E_\alpha) = -\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{j^*\alpha}^{j} - G_{j\alpha}^{j^*}). \tag{1.3}$$

On the other hand, from the definition of $\tilde{\rho}_V$ in Theorem 1, we have

$$\begin{aligned}\tilde{\rho}_V(E_i) =& -\frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_i,E_j,E_j,V)+R(E_i,E_{j^*},E_{j^*},V)) \\ &+ \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}R(E_{j^*},E_j,E_{i^*},V) \\ &- \frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}R(E_i,E_\beta,E_\beta,V),\end{aligned} \tag{1.4}$$

$$\begin{aligned}\tilde{\rho}_V(E_{i^*}) =& -\frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_{i^*},E_j,E_j,V)+R(E_{i^*},E_{j^*},E_{j^*},V)) \\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}R(E_{j^*},E_j,E_{i},V) \\ &- \frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}R(E_{i^*},E_\beta,E_\beta,V),\end{aligned} \tag{1.5}$$

$$\begin{aligned}\tilde{\rho}_V(E_\alpha) =& -\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_\alpha,E_j,E_j,V)+R(E_\alpha,E_{j^*},E_{j^*},V)) \\ &- \sum_{\beta=2m+1}^{n}R(E_\alpha,E_\beta,E_\beta,V).\end{aligned} \tag{1.6}$$

Let $G_{ij}^{k} = g(\nabla_{E_i}E_j, E_k)$ describe the components of covariant differentiation on this frame field. We then have

$$(\nabla V)_i^j = G_{in}^j \quad \text{and} \quad G_{ij}^k = -G_{ik}^j\,.$$

From Lemma 12 in [3], for a unit Killing vector field $V$, the components of the curvature tensor are given by

$$\begin{aligned}R_{jikn} =& -E_i((\nabla V)_j^k) + E_j((\nabla V)_i^k) \\ &+ \sum_{l=1}^{n-1}\{-G_{il}^k(\nabla V)_j^l + G_{jl}^k(\nabla V)_i^l + G_{ij}^l(\nabla V)_l^k - G_{ji}^l(\nabla V)_l^k\}.\end{aligned} \tag{1.7}$$

Using (1.4) and applying (1.7), we obtain

$$\begin{aligned}\tilde{\rho}_V(E_i) =& -\frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{\lambda_i G^j_{ji^*} - \lambda_j G^j_{ij^*} + \lambda_j G^{j^*}_{ji} - \lambda_j G^{j^*}_{ij} \\ &\qquad + E_{j^*}(\lambda_j g_{ij}) + \lambda_i G^{j^*}_{j^*i^*} + \lambda_j G^{j^*}_{ij} - \lambda_j G^j_{j^*i} + \lambda_j G^j_{ij^*}\} \\ &+ \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{-E_{j^*}(\lambda_i g_{ij}) - \lambda_j G^{i^*}_{jj} - \lambda_j G^{i^*}_{j^*j^*} - \lambda_i G^i_{jj^*} + \lambda_i G^i_{j^*j}\} \\ &- \frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}\lambda_i G^\beta_{\beta i^*}\,.\end{aligned}$$

This yields:

$$\begin{aligned}\tilde{\rho}_V(E_i) =& \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{(-1+\lambda_j^2)G^j_{ji^*} + (-1+\lambda_j^2)G^{j^*}_{j^*i^*}\} \\ &+ \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{(-1+\lambda_i^2)G^{j^*}_{ji} + (1-\lambda_i^2)G^j_{j^*i}\} \\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}G^\beta_{\beta i^*} - \frac{1}{1+\lambda_i^2}E_{i^*}(\lambda_i)\end{aligned} \tag{1.8}$$

Similarly, from (1.7), and (1.5), we obtain

$$\begin{aligned}\tilde{\rho}_V(E_i^*) =& -\frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{-E_j(\lambda_i g_{ij}) - \lambda_i G^j_{ji} - \lambda_j G^j_{i^*j^*} + \lambda_j G^{j^*}_{ji^*} \\ &\qquad - \lambda_j G^{j^*}_{i^*j} - \lambda_i G^{j^*}_{j^*i} + \lambda_j G^{j^*}_{i^*j} - \lambda_j G^j_{j^*i^*} + \lambda_j G^j_{i^*j^*}\} \\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{-E_j(\lambda_j g_{ij}) - \lambda_j G^i_{jj} - \lambda_j G^i_{j^*j^*} + \lambda_i G^{i^*}_{jj^*} - \lambda_i G^{i^*}_{j^*j}\} \\ &+ \frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}\lambda_i G^\beta_{\beta i}\,.\end{aligned}$$

This simplifies to become

$$\begin{aligned}\tilde{\rho}_V(E_i^*) =& -\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{(-1+\lambda_j^2)G^j_{ji} + (-1+\lambda_j^2)G^{j^*}_{j^*i}\} \\ &+ \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{(-1+\lambda_i^2)G^{j^*}_{ji^*} + (1-\lambda_i^2)G^j_{j^*i^*}\} \\ &+ \frac{\lambda_i}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}G^\beta_{\beta i} + \frac{1}{1+\lambda_i^2}E_i(\lambda_i)\,.\end{aligned} \tag{1.9}$$

From (1.6), we also have that:

$$\begin{aligned}\tilde{\rho}_V(E_\alpha) &= -\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{-\lambda_j G^j_{\alpha j^*} + \lambda_j G^{j^*}_{j\alpha} - \lambda_j G^{j^*}_{\alpha j} + \lambda_j G^{j^*}_{\alpha j} - \lambda_j G^j_{j^*\alpha} + \lambda_j G^j_{\alpha j^*}\} \\ &= \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G^j_{j^*\alpha} - G^{j^*}_{j\alpha})\end{aligned} \tag{1.10}$$

Comparing (1.1) $\sim$ (1.3) and (1.8) $\sim$ (1.10), we see that (1.1) is not equal to (1.8) and (1.2) is not equal to (1.9). Also (1.3) is not equal to (1.10), that is, it is impossible to obtain Theorem 1.

For this reason we shall, instead, use the definition of $\tilde{\rho}_V(X)$ which is given in Theorem 2. Then we have the relation

$$
\begin{aligned}
\tilde{\rho}_V(E_i) =& \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_i,E_j,E_j,V)+R(E_i,E_{j^*},E_{j^*},V)) \\
&+\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}R(E_{j^*},E_j,E_{i^*},V) \\
&+\frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}R(E_i,E_\beta,E_\beta,V),
\end{aligned}
\tag{1.11}
$$

the relation

$$
\begin{aligned}
\tilde{\rho}_V(E_{i^*}) =& \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_{i^*},E_j,E_j,V)+R(E_{i^*},E_{j^*},E_{j^*},V)) \\
&-\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}R(E_{j^*},E_j,E_i,V) \\
&+\frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}R(E_{i^*},E_\beta,E_\beta,V),
\end{aligned}
\tag{1.12}
$$

and the relation

$$
\begin{aligned}
\tilde{\rho}_V(E_\alpha) =& \sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}(R(E_\alpha,E_j,E_j,V)+R(E_\alpha,E_{j^*},E_{j^*},V)) \\
&+\sum_{\beta=2m+1}^{n}R(E_\alpha,E_\beta,E_\beta,V).
\end{aligned}
\tag{1.13}
$$

Using the Lemma 12 [3] and applying Equations (1.11) – (1.13), we obtain:

$$
\begin{aligned}
\tilde{\rho}_V(E_i) =& \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{\lambda_i G^j_{ji^*}-\lambda_j G^j_{ij^*}+\lambda_j G^{j^*}_{ji}-\lambda_j G^{j^*}_{ij} \\
&\qquad +E_{j^*}(\lambda_j g_{ij})+\lambda_i G^{j^*}_{j^*i^*}+\lambda_j G^{j^*}_{ij}-\lambda_j G^j_{j^*i}+\lambda_j G^j_{ij^*}\} \\
&+\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{-E_{j^*}(\lambda_i g_{ij})-\lambda_j G^{i^*}_{jj}-\lambda_j G^{i^*}_{j^*j^*}-\lambda_i G^i_{jj^*}+\lambda_i G^i_{j^*j}\} \\
&+\frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}\lambda_i G^\beta_{\beta i^*}\,.
\end{aligned}
$$

Consequently

$$\begin{aligned}\tilde{\rho}_V(E_i) =& \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{(1+\lambda_j^2)G_{ji^*}^{j} + (1+\lambda_j^2)G_{j^*i^*}^{j^*}\}\\ &+ \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{(1+\lambda_i^2)G_{ji}^{j^*} - (1+\lambda_i^2)G_{j^*i}^{j}\}\\ &+ \frac{\lambda_i}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}G_{\beta i^*}^{\beta} + \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_{i^*}(\lambda_i).\end{aligned}$$

This yields:

$$\begin{aligned}\tilde{\rho}_V(E_i) =& \frac{\lambda_i}{1+\lambda_i^2}\{\sum_{j=1}^{m}G_{ji^*}^{j} + \sum_{j=1}^{m}G_{j^*i^*}^{j^*} + \sum_{\beta=2m+1}^{n}G_{\beta i^*}^{\beta}\}\\ &+ \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{ji}^{j^*} - G_{j^*i}^{j}) + \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_{i^*}(\lambda_i)\\ =& \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{n-1}G_{ji^*}^{j} + \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{ji}^{j^*} - G_{j^*i}^{j}) + \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_{i^*}(\lambda_i),\end{aligned} \tag{1.14}$$

We continue the computation:

$$\begin{aligned}\tilde{\rho}_V(E_i^*) =& \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{-E_j(\lambda_i g_{ij}) - \lambda_i G_{ji}^{j} - \lambda_j G_{i^*j^*}^{j} + \lambda_j G_{ji^*}^{j^*}\\ &- \lambda_j G_{i^*j}^{j^*} - \lambda_i G_{j^*i}^{j^*} + \lambda_j G_{i^*j}^{j^*} - \lambda_j G_{j^*i^*}^{j} + \lambda_j G_{i^*j^*}^{j}\}\\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{-E_j(\lambda_j g_{ij}) - \lambda_j G_{jj}^{i} - \lambda_j G_{j^*j^*}^{i} + \lambda_i G_{jj^*}^{i^*} - \lambda_i G_{j^*j}^{i^*}\}\\ &- \frac{1}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}\lambda_i G_{\beta i}^{\beta},\end{aligned}$$

so that:

$$\begin{aligned}\tilde{\rho}_V(E_i^*) =& -\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{1}{1+\lambda_j^2}\{(1+\lambda_j^2)G_{ji}^{j} + (1+\lambda_j^2)G_{j^*i}^{j^*}\}\\ &+ \frac{1}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}\{(1+\lambda_i^2)G_{ji^*}^{j^*} - (1+\lambda_i^2)G_{j^*i^*}^{j}\}\\ &- \frac{\lambda_i}{1+\lambda_i^2}\sum_{\beta=2m+1}^{n}G_{\beta i}^{\beta} - \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_i(\lambda_i)\\ =& -\frac{\lambda_i}{1+\lambda_i^2}\{\sum_{j=1}^{m}G_{ji}^{j} + \sum_{j=1}^{m}G_{j^*i}^{j^*} + \sum_{\beta=2m+1}^{n}G_{\beta i}^{\beta}\}\\ &+ \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{ji^*}^{j^*} - G_{j^*i^*}^{j}) - \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_i(\lambda_i)\\ =& -\frac{\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{n-1}G_{ji}^{j} - \sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}(G_{j^*i^*}^{j} - G_{ji^*}^{j^*}) - \frac{1-\lambda_i^2}{(1+\lambda_i^2)^2}E_i(\lambda_i).\end{aligned} \tag{1.15}$$

Finally, we have that

$$\begin{aligned}\tilde{\rho}_V(E_\alpha) &= \sum_{j=1}^{m} \frac{1}{1+\lambda_j^2}\{-\lambda_j G^{j}_{\alpha j^*} + \lambda_j G^{j^*}_{j\alpha} - \lambda_j G^{j^*}_{\alpha j} + \lambda_j G^{j^*}_{\alpha j} - \lambda_j G^{j}_{j^*\alpha} + \lambda_j G^{j}_{\alpha j^*}\} \\ &= -\sum_{j=1}^{m} \frac{\lambda_j}{1+\lambda_j^2}(G^{j}_{j^*\alpha} - G^{j^*}_{j\alpha}). \qquad (1.16)\end{aligned}$$

From Equation (1.1) and (1.14), we see that

$$\frac{1}{f}\omega_V(E_i) - \rho_V(E_i) = \frac{2\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}E_{i^*}(\lambda_j).$$

Since

$$((L_V^{-1}\circ\nabla V)E_i)f = -f\frac{2\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}E_{i^*}(\lambda_j), \qquad (1.17)$$

we have

$$\frac{1}{f}\omega_V(E_i) - \rho_V(E_i) = -\frac{1}{f}((L_V^{-1}\circ\nabla V)E_i)f.$$

Similarly, since

$$((L_V^{-1}\circ\nabla V)E_{i^*})f = f\frac{2\lambda_i}{1+\lambda_i^2}\sum_{j=1}^{m}\frac{\lambda_j}{1+\lambda_j^2}E_i(\lambda_j) \text{ and} \qquad (1.18)$$

$$((L_V^{-1}\circ\nabla V)E_\alpha)f = 0, \qquad (1.19)$$

we have the same results for (1.2) and (1.15) and for (1.3) and (1.16), respectively. This completes the proof of the Theorem 2. □

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